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THEK-RRAM

**Real-Record Attenuation Method:
Structural Evolution and Second-Generation Law**

From the near-source-tail limitation of the first generation to magnitude-dependent spatial attenuation

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Abstract

This paper documents the second stage in the development of Moncayo Theurer's Real-Record Attenuation Method (THEK-RRAM). Paper I presented the instrumental database, the CMAR as the envelope of maximum recorded accelerations, the thirteen historical attenuation laws used as references, and the derivation of the four first-generation laws. Paper II retains that comparative framework and focuses on the evolution that led to the second-generation law. In this paper, 1G denotes the first-generation THEK-RRAM attenuation laws, whereas 2G denotes the second-generation THEK-RRAM attenuation law.

The need to evolve the formulation emerged when the first-generation laws were contrasted with new evidence, particularly during the test performed using the 2023 Türkiye earthquake, Mw 7.8.

The first-generation equations (1G) shared a fixed spatial attenuation rate: when the near-source zone was fitted, the calculated values could remain too high at large distances; when the tail of the curve was fitted, the near-source response could become too low.

Moncayo Theurer then reopened the curve-calculation process using FGET-VRR-NFF and added a new level of precision to the magnitude dependence by incorporating M^2 and the MD interaction.

The evolution produced an equation in which the amplitude depends on a nonlinear structure in M and the spatial slope is no longer universal, but instead depends on magnitude. The paper reconstructs this genealogy, compares the thirteen historical laws and the four 1G laws, explains the role of the 2023 Türkiye earthquake as a milestone in the readjustment, and finally summarizes the subsequent academic validation by Ramón Casquete and Zamora (2023), who evaluated eighteen formulations against fifty control earthquakes [2].

Keywords: THEK-RRAM; second-generation law; FGET; THEK-VRR; NFF; seismic attenuation; CMAR; PGA; magnitude; distance; formula evolution.

1. Continuity with Paper I

The starting point of this work consists of the four first-generation laws developed by Moncayo Theurer. They all share a general structure that primarily relates acceleration to magnitude and distance.

$$a_{1G} = A e^{BM} e^{-0.005D}$$

This structure has a direct mathematical consequence: for any value of M, the logarithmic slope with respect to D is the same.

$$\frac{\partial \ln(a_{1G})}{\partial D} = -0.005 \text{ km}^{-1}$$

Therefore, the 1G curves shift vertically with magnitude but retain the same exponential rate of decrease with distance. This property, useful because of its simplicity, also became the feature that had to be reconsidered when new evidence was incorporated.

1.1 Historical relationships used as comparative references

The following thirteen relationships were used in the Miyagi comparative study and in subsequent work within the THEK-RRAM research line [3,4,19].

Table 1. Thirteen historical attenuation laws evaluated and their mathematical expressions [3].

Attenuation law	Equation
Donovan 1 (D1)	$a_h = 1320e^{0.580M}(R + 25)^{-1.520}$
Donovan 2 (D2)	$a_h = 1080e^{0.500M}(R + 25)^{-1.320}$
Donovan 3 (D3)	$\ln(a_h) = 6.98 + 0.50M - 1.25 \ln(R + 25)$
McGuire (MG)	$a_h = 472.3e^{0.640M}(R + 25)^{-1.301}$
Gómez, Ordaz and Tena (GOT1)	$\ln(a_h) = 2.308 + 0.780M - 0.844 \ln(R) - 0.004R$
Gómez, Ordaz and Tena (GOT2)	$\ln(a_h) = 1.237 + 1.519M - 0.0313M^2 - 0.844 \ln(R) - 0.004R$
Ordaz, Jara and Singh (OJS)	$\log_{10}(a_h) = 1.760 + 0.30M - \log_{10}(R) - 0.00310R$

Singh (Si)	$\log_{10}(a_h) = -0.148 + 0.623M - \log_{10}(R) - 0.00320R$
Grases 1 (G1)	$\ln(a_h) = 3.75 + 0.470M - 0.570[\ln(R + 10) - 0.670]$
Grases 2 (G2)	$\ln(a_h) = 3.75 + 0.470M - 0.570[\ln(R + 10) + 0.670]$
Aguiar 1 (Ag.1)	$\ln(a_h) = 6.35 + 0.99M - 1.76 \ln(R + 40) - 0.6$
Aguiar 2 (Ag.2)	$\ln(a_h) = 6.35 + 0.99M - 1.76 \ln(R + 40) + 0.6$
Sarangoni (Sa)	$\ln(a_h) = 8.54 + 0.570M - 1.73 \ln(R + 60)$

These equations constitute the historical comparison framework.

1.2 The four first-generation laws of Moncayo Theurer

The four first-generation expressions share the same spatial attenuation rate of -0.005 km^{-1} . They differ in base level and magnitude sensitivity, but they retain the same exponential slope with distance. This common feature is the mathematical point that was subsequently re-examined [1].

Table 2. Four first-generation laws of Moncayo Theurer [1].

Law	Expression	Feature
Original Curve (C0)	$a_{C0} = 36 \cdot e^{0.52M} \cdot e^{-0.005D}$	Base first-generation formulation.
Variant 1 (V1)	$a_{V1} = 12 \cdot e^{0.65M} \cdot e^{-0.005D}$	Greater exponential sensitivity to M.
Variant 2 (V2)	$a_{V2} = 60 \cdot e^{0.45M} \cdot e^{-0.005D}$	Higher scale A and lower exponent of M.
Reduced Original	$a_{Reb} = 18 \cdot e^{0.52M} \cdot e^{-0.005D}$	50% reduction of factor A relative to C0.

The Original Curve, Variant 1, Variant 2, and Reduced Original allowed different balances between initial level and magnitude-dependent growth to be explored. However, all inherited the same spatial attenuation rate of -0.005 km^{-1} ; therefore, none of them could independently change the shape of the tail as a function of M.

2. The near-source-tail problem evidenced by Türkiye 2023

It was observed that when the coefficients were oriented toward reproducing the high accelerations in the near-source zone, the curves could retain values that were too high in the far field; when the fit was shifted toward large distances, the calculated response near $D \approx 0$ could become too low. The problem was not

merely one of scale: it was a limitation of the functional form, or of the shape of the curve.

The test using the 2023 Türkiye earthquake, Mw 7.8, revealed this situation. The available PGA records were compared with the four 1G formulations, and the event was used as a milestone for a new adjustment stage [15]. The comparison showed that maintaining a single spatial slope for the entire curve made it difficult to represent the near field and the tail simultaneously.

Türkiye provided the evidence that made the mismatch in values visible and forced the mathematical structure to be reopened.

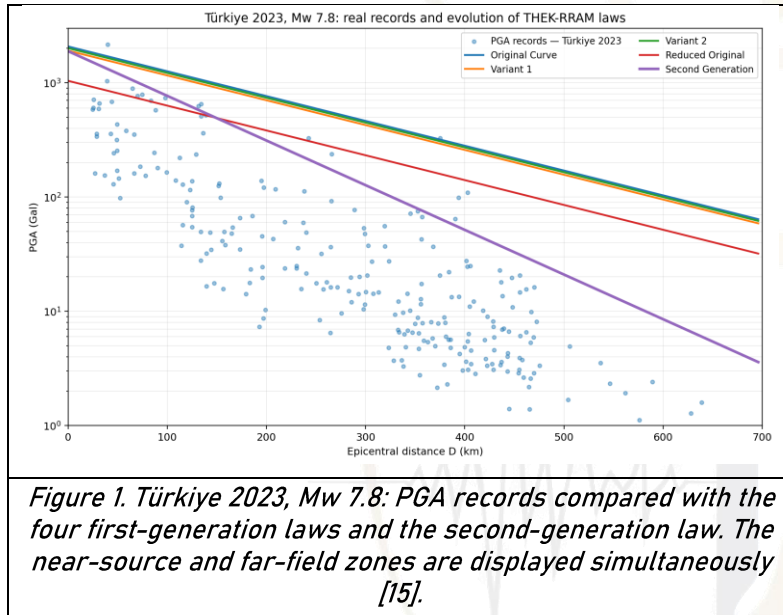


Figure 1 shows how the first-generation curves agree very well with the 2023 Türkiye Mw 7.8 earthquake. This result was not obtained through a direct fit: the first-generation formulas identified the maximum earthquake accelerations in the near-source zone; however, as previously observed, beyond about 400 km these laws move away from the recorded values, indicating that the shape of the

curve had to change so that the values would decrease. The figure also shows that Variants 1 and 2 are more extreme and higher, while the Original Curve produces values closer to the recorded values, subject to the limitation indicated above.

The figure also shows the second-generation curve, and it can be seen that this curve fits both near-source and far-field values, which was the objective of the investigation.

The problem of values corresponding to distances far from $D = 0$, which were too high, gave rise to the second FGET-THEK-VRR-NFF cycle.

3. Reapplication of FGET–THEK–VRR–NFF

The central methodological feature of this stage is that the first generation was not treated as a closed formula. The existing equation re-entered the development cycle. In the current terminology, FGET reopens the functional structure; THEK–VRR focuses the exploration on terms capable of resolving the observed deficiency; and NFF reorganizes and nests the dependencies among coefficients and variables [5–7].

In this second application, a new layer of precision was introduced into the magnitude dependence. In 1G, M acted primarily through a linear term in the exponent, BM.

The new calculation allowed the magnitude dependence to acquire curvature through M^2 and, in addition, allowed M to interact with distance through MD. The M^2 term expands the ability to adjust how amplitude grows with magnitude; the MD term allows the spatial slope to cease being fixed. Together, both terms form the second nesting level that corrects the near-source–tail trade-off.

NFF has consistently been used to introduce new variables into the formula. In this case, it was applied a second time to the same variable M , thereby refining the calculation.

The complete procedure for exploring, weighting, and selecting candidate formulations is part of the THEK–RRAM know-how. This paper presents the mathematical genealogy required to understand the transition from the first-generation attenuation law to the second-generation law. For specific applications, calibrations, or detailed implementation, the method should be discussed with the author at research@hithek.com.

4. Second-generation law

The second-generation attenuation law for calculating accelerations is expressed, in Gal, as:

$$a_{2G} = (105 - 5M) e^{(0.235M + 0.025M^2)} e^{(-0.0285D + 0.0025MD)}$$

where a_{2G} is the acceleration predicted by the second-generation law in Gal (cm/s^2), M is the earthquake magnitude, and D is the epicentral distance in km.

The M^2 term modifies the amplitude dependence on magnitude, while the MD term couples magnitude and distance.

Because D is expressed in km, the coefficients associated with D have dimensions compatible with km^{-1} so that the exponent remains dimensionless.

The equation can be rearranged to make the spatial attenuation rate explicit:

$$a_{2G} = (105 - 5M) e^{(0.235M + 0.025M^2)} e^{D(-0.0285 + 0.0025M)}$$

$$k(M) = -0.0285 + 0.0025M$$

The M^2 term introduces a new layer of precision in the treatment of magnitude: it introduces a nonlinear dependence that was not present in the four 1G laws. In turn, the MD interaction causes magnitude to also modify the decay with D . Therefore, the second-generation (2G) solution is a doubly nested structure with respect to M .

5. What changes relative to the first generation

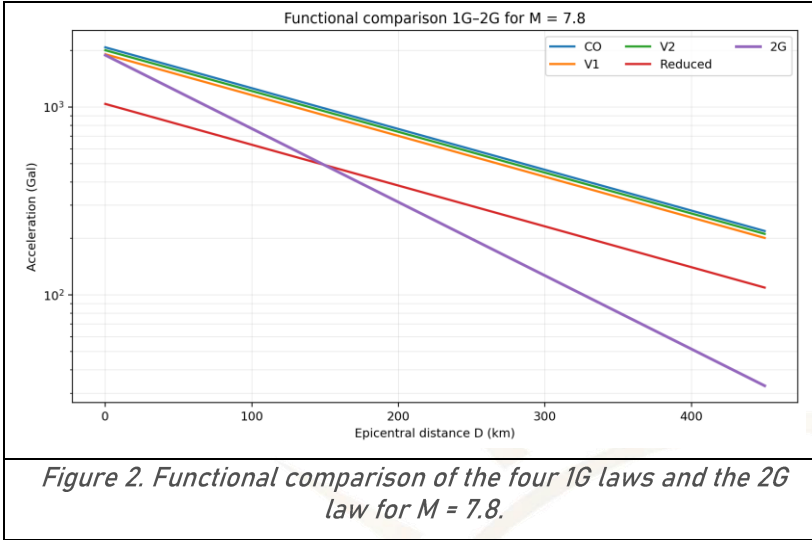
The difference between generations can be seen directly in the logarithmic derivative with respect to distance:

$$\begin{aligned} \text{1G: } \frac{\partial \ln(a_{1G})}{\partial D} &= -0.005 \\ \text{2G: } \frac{\partial \ln(a_{2G})}{\partial D} &= -0.0285 + 0.0025M \end{aligned}$$

In the first generation (1G), all magnitudes share the same slope. In the second generation (2G), the slope changes with M .

The MD term causes magnitude and distance to cease acting in a completely separable manner.

Through the double application of NFF, the second-generation law is presented as the most elaborate and most effective formulation in the family. This assessment is based on the fact that 2G corrects the rigidity of the fixed slope and can represent both the near-source and far-field zones with a single structure, as shown in the figure.



6. Role of the CMAR in the second generation

The evolution to 2G does not change the fundamental definition of the CMAR used by Moncayo Theurer. The CMAR represents the envelope of the maximum recorded accelerations as a function of distance for each earthquake or magnitude class.

It is an upper empirical reference, not an average curve.

THEK-RRAM does not seek to force the attenuation law to coincide systematically with the CMAR.

The historical development criterion places the formulation approximately below that envelope, in a range on the order of 75–80% of the CMAR, seeking a balance between avoiding severe underestimation and not transferring every observed maximum directly into design.

$$CPAR < a_{\text{THEK-RRAM}} < \text{CMAR}$$

$$a_{\text{THEK-RRAM}} \approx (0.75-0.80)a_{\text{CMAR}}$$

The second generation retains this philosophy but seeks to represent the evolution of demand with distance more consistently by allowing the slope to vary with magnitude.

7. Sensitivity of the equation to magnitude

The dependence on M is not limited to vertical scaling. Differentiating the logarithm of the equation with respect to magnitude gives:

$$\frac{\partial \ln(a_{2G})}{\partial M} = -\frac{5}{105 - 5M} + 0.235 + 0.05M + 0.0025D$$

The $0.0025D$ term shows that sensitivity to magnitude also increases with distance. This property is consistent with the intention that large events retain a spatial influence different from that of smaller events. It is a mathematical property of the formulation.

8. Scope of the methodological originality

The specific contribution of this work is the integrated use of the FGET-VRR-NFF architecture to further evolve an equation created by the same triad after a need for refinement was identified.

In this sense, the 1G→2G transition constitutes an internal demonstration of the evolutionary-formula concept: the equation is not regarded as definitive simply because it has been calibrated once, but may return to the discovery cycle and modify its structure.

This principle connects THEK-RRAM with contemporary lines of symbolic regression and physics-informed equation discovery, while retaining its own architecture and a specific engineering objective [8–13].

9. Use, traceability, and know-how

For professional applications, adaptation to regional databases, definition of the domain of use, comparison with a specific code, or integration into other THEK modules, our research team should be contacted at research@hithek.com.

This interaction will preserve traceability regarding the version of the equation used, the units, the selection criterion, and the particular conditions of each application.

10. Limitations and future work

The second generation maintains a deliberately reduced parameterization in M and D and does not explicitly include all site, fault-mechanism, rupture, or path effects used in contemporary GMMs [14]. Paper I explained that, as a scientific strategy in the effort to develop an energy-based design method, the attenuation law should remain simple [17,18].

These limitations define its domain of verification.

11. Subsequent academic validation: thesis by Ramón Casquete and Zamora (2023)

The subsequent academic validation was carried out in the thesis “Efficiency of Attenuation Laws against Real Earthquakes,” developed by Adriana Lissette Ramón Casquete and Ingrid Zamora at the University of Guayaquil in 2023 [2]. The study selected fifty control earthquakes distributed across magnitude ranges, with a minimum of approximately twenty records per event, and evaluated eighteen formulations: the thirteen historical laws and the five laws in the Moncayo Theurer family, including the second generation.

The authors compared the predictions with real records using percentage errors relative to PGA and the CMAR, and synthesized performance through a decision matrix. Five representative events show that performance depends on magnitude and on the evaluation criterion.

Table 3. Five representative earthquakes from the validation by Ramón Casquete and Zamora (2023) [2].

Control earthquake	Mw	Relevant result	Interpretation for 2G
California, USA	4.7	≈2% error relative to PGA	Strong 2G performance.
Türkiye	5.8	Grases 1 / Aguiar 1 performed better in this case	2G is not dominant.
Alaska, USA	6.1	2G ≈22%; GOT2 ≈29%	Favorable result for 2G.
Pedernales, Ecuador	7.8	2G ≈3% relative to PGA	Very good point estimate.
Chile	8.8	2G ≈8%; V2 ≈7% relative to CMAR	2G remains competitive at high magnitude.

This thesis compares the 18 attenuation laws in order to determine which one best represents the accelerations actually recorded during earthquakes. The overall conclusion of the thesis does not establish a single winner.

For the approximate Mw 4–6 range, however, the second-generation law and GOT2 achieved a technical tie by ranking among the most accurate formulations for predicting accelerations in the 50 earthquakes analyzed. For magnitudes from Mw 7 upward, GOT2 and Variant 1 were the most effective formulations for estimating maximum accelerations across the 50 earthquakes studied. A decision-matrix methodology was used to determine the best combinations [2].

The second-generation law (2G) showed approximately 2% error relative to PGA for California Mw 4.7; about 22% for Alaska Mw 6.1 compared with approximately 29% for GOT2; about 3% relative to PGA for Pedernales Mw 7.8; and about 8% relative to PGA for Chile Mw 8.8.

For the latter event, Variant 2 reached approximately 7% error. In the Türkiye Mw 5.8 case included in the thesis, some historical laws—Grases 1 and Aguiar 1—outperformed the RRAM formulations, confirming the natural mathematical sensitivity of a model of this type [2].

The thesis confirms competitive performance and very favorable results for the second-generation attenuation law (2G) in several events, while also showing that GOT2, Variant 1, Variant 2, or other relationships may perform better under particular ranges and criteria.

12. Conclusions

1. The second generation arose from a structural need identified in the four first-generation laws (1G): all shared a fixed spatial attenuation rate and therefore the same basic form of decay with D .
2. The Türkiye 2023 test, Mw 7.8, emphasized the need for a formulation that maintains a more pronounced decay at distances far from the epicenter.
3. The second use of the FGET-THEK-VRR-NFF sequence for refinement of the variable M opens interesting research directions. This is considered a second level of nesting on magnitude: M^2 added curvature to the amplitude dependence, and MD allowed the spatial attenuation rate to depend on M .
4. The resulting second-generation law retains an explicit form in M and D , but replaces the fixed 1G spatial slope with $k(M) = -0.0285 + 0.0025M$, refining the form that motivated the evolution.
5. CMAR remains the envelope of maximum recorded accelerations; THEK-RRAM seeks a balanced upper demand below that envelope, on the order of 75–80%, but above average values, consistent with the extreme-event criterion.

6. The thesis by Ramón Casquete and Zamora (2023), based on fifty control earthquakes and eighteen laws, provides independent validation: the second-generation attenuation law (2G) obtained particularly strong results in most events; other laws that stood out were GOT2, Variant 2, and the Original Curve.

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