

THEK RESEARCH INSTITUTE

RRAM

• PAPER I

THEK Real-Record Attenuation Method

FGET-THEK-VRR-NFF architecture and development

*From the exploration of instrumental patterns to the structural evolution
of explicit equations*

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THEK Research Institute · August 2026

Paper I: development of the first four attenuation laws.

Paper II: evolution and the Second-Generation Law.

Abstract

This paper presents the methodological and mathematical core of the THEK Real-Record Attenuation Method (THEK-RRAM), developed by Marcelo Moncayo Theurer as a continuation of a line of research initiated through comparisons between historical attenuation laws and real seismic records.

Paper I focuses on the construction of the first four formulations: how instrumental evidence, the Maximum Recorded Acceleration Curve (CMAR), and the architecture currently formalized as FGET-THEK-VRR-NFF led to explicit first-generation equations. The terms FGET, THEK-VRR, and NFF correspond to the current formalization of the process [25–28].

THEK-RRAM starts from instrumental records and uses magnitude M , epicentral distance D , and peak ground acceleration a as widely available operational variables.

CMAR —the Maximum Recorded Acceleration Curve— serves as an upper empirical reference. However, the design equation sought by Moncayo Theurer is not intended to reproduce the absolute maximum, but rather to lie deliberately below it, at approximately 75–80% of CMAR, so that the mathematical outcome remains consistent with an extreme-value criterion while retaining a balanced level of demand.

The paper reconstructs the first generation of laws, which combine magnitude and distance, and presents its four expressions: Original Curve, Variant 1, Variant 2, and Reduced Original. The objective is to provide sufficient traceability of the development: base events, instrumental records, construction of CMAR envelopes, identification of the functional pattern, and progressive derivation of the equations.

Operational implementation details can be discussed with the author for specific applications.

Keywords: THEK-RRAM; FGET; THEK-VRR; NFF; attenuation laws; ground-motion model; PGA; CMAR; magnitude; distance; empirical equations; formula evolution.

1. Introduction: from diagnosis to the construction of a new relationship

Thirteen historical attenuation laws were compared with records from instrumented events of approximate magnitudes 5, 6, 7, and 9, and significant discrepancies were found in the representation of the upper acceleration branch (CMAR), especially for the higher-magnitude events [29,30,34,35].

Once the problem has been identified, the question becomes: how can a new mathematical relationship be constructed from the evidence?

In GMM development, international practice has evolved from relatively simple magnitude–distance relationships toward models that incorporate numerous source, path, and site terms, together with variability and mixed-effects models [1–12,38–44].

This evolution has increased descriptive capability, but also parametric complexity and dependence on metadata. Recent reviews note that the growing number of parameters can introduce correlations, trade-offs, and overfitting risk; therefore, parsimony and interpretability remain legitimate objectives and are among the aims of this work [45].

In parallel, this formulation is part of a broader research program aimed at developing an energy-based structural design methodology [32,33]. For that purpose, retaining a compact and operational attenuation law is valuable. This research line is also connected with the author's previous work on seismic characterization and recurrence in Ecuador [36,37]. Likewise, equation discovery from data through genetic algorithms and physics-informed approaches has become increasingly demanding over time [13–18,24].

THEK-RRAM is positioned within this space. The process must be able to generate alternatives, reduce the space of variables and terms, organize dependencies in a nested manner, and, finally, reopen a completed equation when new evidence reveals a deficiency. In its current formalization, these functions are identified as FGET, VRR, and NFF [25–28].

2. Objective and scope of Paper I

The objective is to document the methodological architecture of THEK-RRAM and mathematically reconstruct the transition from instrumental observations to the four first-generation laws.

The paper focuses on four aspects: (i) the real earthquake and record database that supported the development; (ii) the role of CMAR as an envelope of maxima; (iii) the function of FGET, THEK-VRR, and NFF in pattern identification, reduction, and nesting; and (iv) the mathematical genealogy of the Original Curve, Variant 1, Variant 2, and Reduced Original.

The evolution toward the Second Generation is developed in a separate paper.

3. Empirical basis and minimum-information principle

THEK-RRAM uses seismic magnitude M , epicentral distance D (km), and peak ground acceleration a (Gal) as its basic operational variables.

The choice of seismic input follows a methodological decision: maximize the number of available records. At this stage of development, the author gives priority to a broad and homogeneous instrumental database so that the mathematical procedure operates on the largest possible number of real observations.

The author also seeks a broadly applicable formulation that can begin with a minimum set of variables because it forms part of a wider research line that requires compact and operational results. At later stages, these formulas can be enriched with site effects, fault mechanism, or path effects, which remain within the future research line [4–9,38,39].

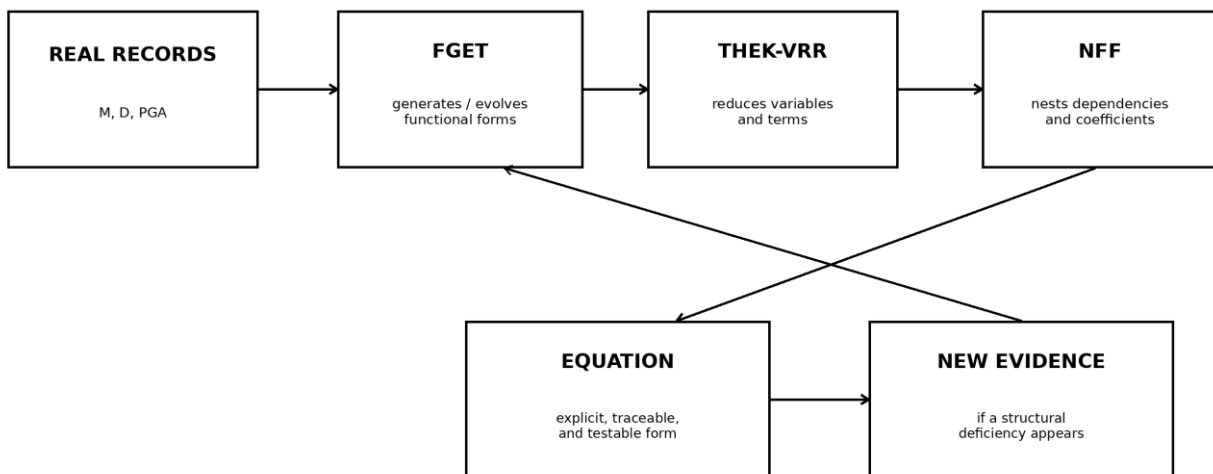
Nevertheless, contemporary development continues to seek simple, interpretable models that remain suitable for extrapolation, precisely because an indiscriminate expansion in the number of proxies and coefficients can also create identification and overfitting problems [45].

THEK-RRAM addresses this issue while preserving simplicity.

Symbol	Variable	Use in THEK-RRAM
CMAR	Maximum Recorded Acceleration Curve	Upper curve or envelope constructed from observed maxima.
CPAR	Average Recorded Acceleration Curve	Central curve, calculated for comparative purposes

4. Integrated THEK-RRAM architecture

THEK-RRAM methodological architecture



The completed equation may return to the cycle when new records reveal a limitation.

Figure 1. Conceptual architecture of THEK-RRAM. Multiple FGET-VRR-NFF cycles may be used [25–28].

The architecture can be expressed as follows: FGET opens the solution space and allows the formula to be treated as an evolutionary structure; VRR takes the data, generates variables, ranks them, and reduces the set; NFF organizes the surviving relationships through recursive coefficient nesting. The output is an explicit, understandable equation fitted to real data. If later records reveal a deficiency in the resulting formula, the

equation can re-enter the FGET-VRR-NFF process to produce a more refined formulation.

For the first time within THEK, FGET-VRR-NFF is used to nest successive levels of mathematical precision in the development of these laws.

4.1 FGET: the formula as an evolutionary structure

Formula Genomics and Evolution Theory (FGET) formalizes the idea that an equation has an internal organization—a “genome” of operators, terms, and relationships—that can be reconstructed, compared, and modified [25].

Applied to engineering, the important point is not to adopt the entire philosophical ontology of FGET literally, but rather its operational consequence: functional form is itself a variable of the problem.

An equation that fits one database may cease to be adequate when the observed pattern changes or when a previously unrepresented domain is incorporated.

This idea has parallels with symbolic regression and the identification of governing equations from data. Schmidt and Lipson showed that explicit mathematical laws can be distilled through expression search [14]; Brunton, Proctor, and Kutz developed SINDy to identify sparse dynamical structures [15]; and Udrescu and Tegmark proposed AI Feynman to discover interpretable formulas [16]. In earthquake engineering, Tavakoli and Pezeshk used a hybrid genetic algorithm to estimate GMPEs and allowed changes in functional form [13]; Chen et al. developed a physics symbolic learner using NGA-West2 [17], and Liu et al. combined symbolic regression with random-effects regression [18]. These studies follow paths that are parallel to, but not the same as, the THEK architecture.

4.2 THEK-VRR: reduction of the variable and term space

THEK-VRR (Variable Reduction & Ranking) performs a function that must be distinguished from simple record filtering. In the current methodological interpretation of THEK Research Institute, VRR reduces the parametric space: it identifies variables, terms, or combinations whose contribution is insufficient relative to the objective, and retains the elements that modify the equation's descriptive capability in a relevant and stable manner.

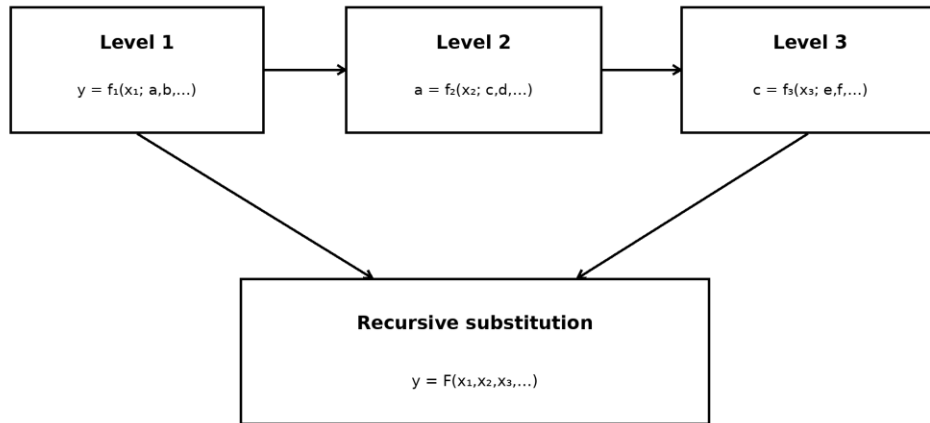
In other applications by the author, such as THEK-SPOT, the FGET→VRR→NFF chain is publicly documented, and VRR acts as a variable-reduction stage before final construction [27].

General variable-selection theory, LASSO, information criteria, and model selection provide an academic framework compatible with this concern for parsimony [19–23], although THEK-VRR has its own selection logic.

$$G_j = J_{\text{ref}} - J(+x_j)$$

Where: G_j = gain associated with incorporating variable or term x_j ; J_{ref} = error function of the reference model; $J(+x_j)$ = error function after incorporating x_j .

4.3 NFF: recursive nesting and coefficient traceability

NFF: recursive nesting of coefficients

Each level explains how a new variable modifies the coefficients of the previous level.

Figure 2. Conceptual scheme of the Nested Formula Framework [26].

NFF is the operational component that makes it possible to construct a multivariable relationship [26]. Recursive substitution ultimately reconstructs a multivariable function while preserving the genealogy of each coefficient.

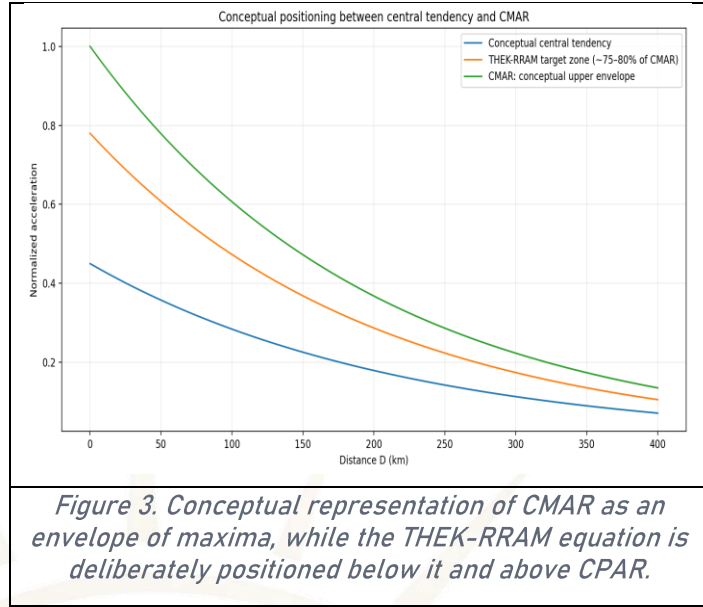
The methodological advantage sought by Moncayo Theurer is traceability of the development.

This idea may be considered parallel to mixed-effects models, which decompose between-event and within-event variability through random effects and are fundamental to probabilistic GMMs [10–12].

NFF addresses a different question: how to progressively construct an explicit functional structure. The two approaches can coexist; the recent combination of symbolic learning with random effects is complementary [18].

5. CMAR as an upper reference and the balance criterion

CMAR —the **Maximum Recorded Acceleration Curve**— represents the envelope of the maximum acceleration values observed for each earthquake [29,30,34,35]. It is not an average curve. Its function is to construct an upper empirical reference, based on real records, against which both historical relationships and new formulations can be compared.



The methodological principle of this work is to construct an equation that remains sufficiently close to the upper branch to avoid severe underestimation, but without attempting to reproduce every isolated maximum.

$$a_{\text{THEK}}(D, M) \approx (0.75 - 0.80) \cdot a_{\text{CMAR}}(D, M)$$

Where:

a_{THEK} = acceleration level represented by the target equation;

a_{CMAR} = acceleration of the CMAR envelope for the same combination of magnitude and distance.

The target therefore corresponds to a balanced demand level: above the central tendency and below the absolute maximum. Its engineering rationale is to balance safety and economy.

In THEK-RRAM, the objective at this stage is not to reproduce the absolute maximum, but to establish a balanced upper demand relative to the CMAR envelope.

5.1 Instrumental basis that led to the first formulations

The development used instrumented earthquakes representing four magnitude levels and having a substantial number of accelerographic records: Chino Hills, California, Mw 5.4 (413 records); Northridge, California, Mw 6.4 (approximately 310 records in the working database); Miyagi, Japan, Mw 7.1 (136 records in the published article); and Tohoku, Japan, Mw 9.0 (500 records). This sequence made it possible to observe acceleration–distance behavior from moderate magnitudes to a great earthquake [29,30,34,35].

For each event, the maximum acceleration values recorded by the stations were plotted as a function of epicentral distance. For each dataset, CMAR was identified as the envelope of the observed maxima. This step was decisive: instead of relying only on an average trend, the analysis followed the upper demand branch actually present in the records.

When the envelopes for the four magnitude levels were compared, the decrease with distance was found to be representable in compact form by exponential functions.

The problem then became one of recognizing which portion of the structure was repeated among events and which portion changed with magnitude.

In the current terminology of the method, FGET identifies and opens the candidate functional families; THEK-VRR reduces the alternatives and retains relationships that provide useful information; and NFF organizes the surviving dependencies in nested form. The result of this process was a family of explicit equations in which magnitude controls the acceleration level and distance controls its spatial attenuation.

Before comparing the real records with the historical relationships, the thirteen attenuation laws used in the Miyagi study were considered. Their expressions, reproduced from Moncayo Theurer et al. (2016) [29], are summarized below.

Table 1. Thirteen historical attenuation laws evaluated and their mathematical expressions [29].

Attenuation law	Equation
Donovan 1 (D1)	$a_h = 1320e^{0.580M}(R + 25)^{-1.520}$
Donovan 2 (D2)	$a_h = 1080e^{0.500M}(R + 25)^{-1.320}$
Donovan 3 (D3)	$\ln(a_h) = 6.98 + 0.50M - 1.25 \ln(R + 25)$
McGuire (MG)	$a_h = 472.3e^{0.640M}(R + 25)^{-1.301}$
Gómez, Ordaz and Tena (GOT1)	$\ln(a_h) = 2.308 + 0.780M - 0.844 \ln(R) - 0.004R$
Gómez, Ordaz and Tena (GOT2)	$\ln(a_h) = 1.237 + 1.519M - 0.0313M^2 - 0.844 \ln(R) - 0.004R$
Ordaz, Jara and Singh (OJS)	$\log_{10}(a_h) = 1.760 + 0.30M - \log_{10}(R) - 0.00310R$
Singh (Si)	$\log_{10}(a_h) = -0.148 + 0.623M - \log_{10}(R) - 0.00320R$
Grases 1 (G1)	$\ln(a_h) = 3.75 + 0.470M - \log_{10}(R) - 0.00320R$
Grases 2 (G2)	$\ln(a_h) = 3.75 + 0.470M - 0.570[\ln(R + 10) + 0.670]$
Aguar 1 (Ag.1)	$\ln(a_h) = 6.35 + 0.99M - 1.76 \ln(R + 40) - 0.6$
Aguar 2 (Ag.2)	$\ln(a_h) = 6.35 + 0.99M - 1.76 \ln(R + 40) + 0.6$
Sarangoni (Sa)	$\ln(a_h) = 8.54 + 0.570M - 1.73 \ln(R + 60)$

Where: a_h is the maximum horizontal acceleration (cm/s² or Gal), M is earthquake magnitude, and R is epicentral distance in km. The equations are preserved using the notation of the source article [29].

5.2 Input for determining the laws

Figure 4 brings together the historical acceleration–distance curves for the events that served as the basis for the first stage of development. Each panel shows the distribution of recorded accelerations as a function of epicentral distance, making it possible to observe how acceleration changes across the different cases and providing the input used to initiate the FGET, VRR, and NFF method.

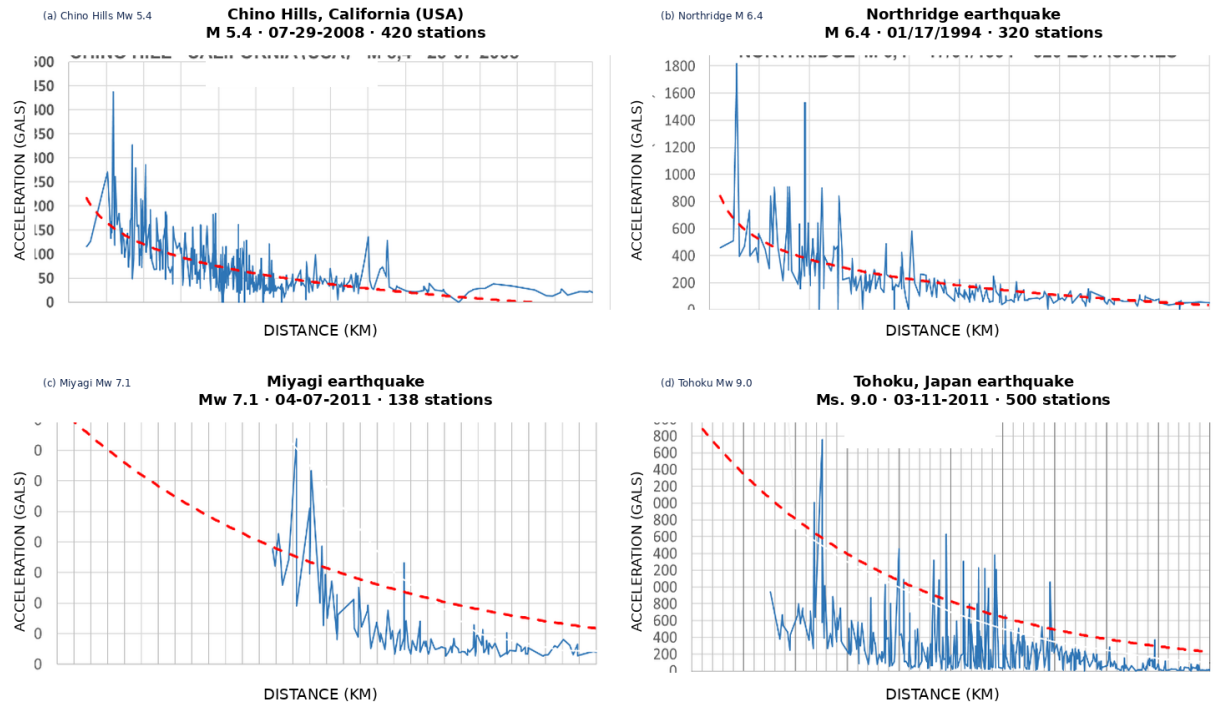


Figure 4. Historical data for the base earthquakes used in method development: Chino Hills (Mw 5.4), Northridge (Mw 6.4), Miyagi (Mw 7.1), and Tohoku (Mw 9.0).

5.3 Significant relative error

Figure 5 shows the error produced by values calculated with historical attenuation-law formulas relative to the real behavior observed for a strong reference earthquake, providing evidence of the need to update the attenuation relationships. As can be seen, most of the attenuation laws tested show relative errors greater than 80% with respect to the earthquake's actual accelerations. The laws that fall below this error level for the Mw 7.1 earthquake are Gómez/Ordaz/Tena 2 (GOT2), Grases 2, and Aguiar 2 over part of the analyzed range.

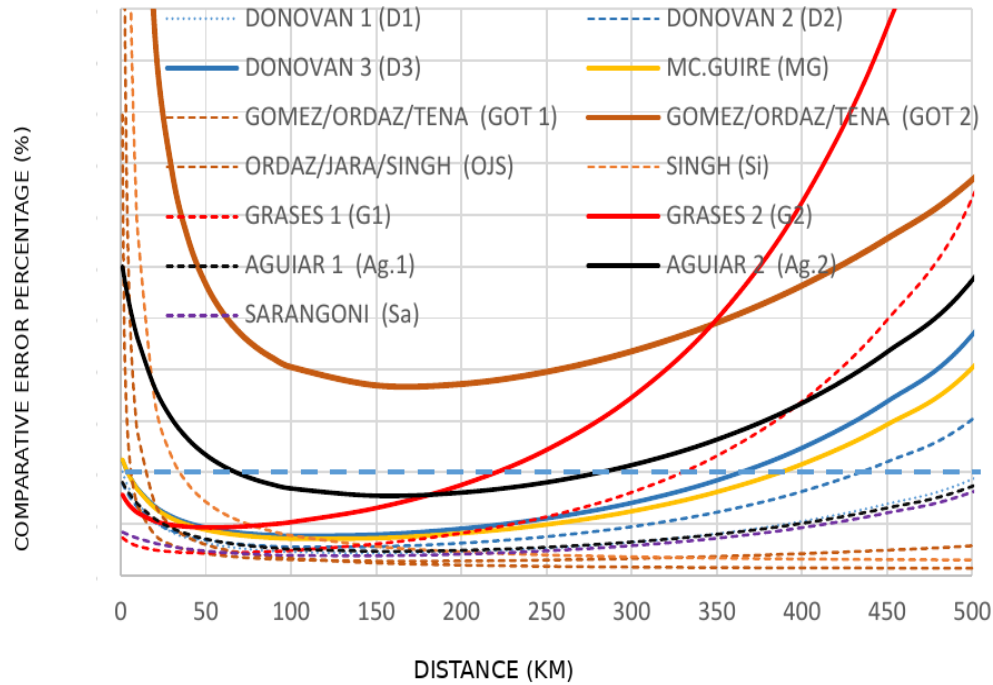


Figure 5. Relative error of the formulations evaluated for the Miyagi earthquake (Mw 7.1).

5.4 Calculation efficiency of the historical laws

Figure 6 shows the difference between the actual maximum values of an Mw 7.1 earthquake and the accelerations calculated with the historical laws, which at the time further emphasized the need to update them.

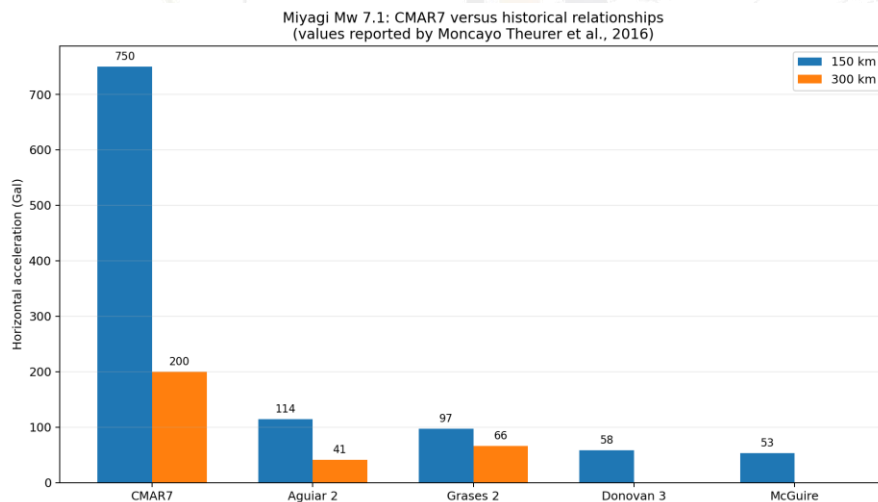


Figure 6. Comparison of Miyagi CMAR7 with the accelerations predicted by historical laws at 150 km and 300 km.

6. Construction of the first-generation laws

From the empirical curves constructed by magnitude class, the author observed that an exponential form could compactly represent acceleration behavior with distance.

In the first generation, a fixed spatial attenuation rate was adopted and the acceleration level was made to depend primarily on magnitude. The result can be written in a general form:

$$a_{1G} = A \cdot e^{BM} \cdot e^{-0.005D}$$

Where:

a_{1G} = acceleration predicted by a first-generation law, in Gal;

A = scale coefficient of the law;

B = magnitude-sensitivity coefficient; M = earthquake magnitude;

D = epicentral distance in km;

A GMPE expressed in logarithmic form can be algebraically transformed into an exponential form of the predicted variable.

The distinctive feature of the Moncayo Theurer laws lies in their specific parameterization, in the way it was obtained from the real curves, and in the methodological genealogy leading to their coefficients.

Table 2. First-generation attenuation laws of Moncayo Theurer.

Law	Expression	A	B	Feature
Original Curve (C0)	$a_{C0} = 36 \cdot e^{0.52M} \cdot e^{-0.005D}$	36	0.52	Base first-generation formulation.
Variant 1 (V1)	$a_{V1} = 12 \cdot e^{0.65M} \cdot e^{-0.005D}$	12	0.65	Greater exponential sensitivity to M.
Variant 2 (V2)	$a_{V2} = 60 \cdot e^{0.45M} \cdot e^{-0.005D}$	60	0.45	Larger scale A and smaller exponent of M.
Reduced Original	$a_{Reb} = 18 \cdot e^{0.52M} \cdot e^{-0.005D}$	18	0.52	50% reduction of factor A relative to C0.

6.1 Original Curve

$$a_{CO} = 36 \cdot e^{0.52M} \cdot e^{-0.005D}$$

Where:

a_{CO} = acceleration of the Original Curve in Gal;

M = magnitude;

D = epicentral distance in km.

This expression retains a constant spatial attenuation rate of -0.005 km^{-1} for all magnitudes and increases exponentially with M .

6.2 Variant 1

$$a_{V1} = 12 \cdot e^{0.65M} \cdot e^{-0.005D}$$

Where:

a_{V1} = acceleration of Variant 1 in Gal;

M = magnitude;

D = epicentral distance in km.

Relative to CO , $V1$ decreases the scale factor and increases the sensitivity of amplitude to M .

6.3 Variant 2

$$a_{V2} = 60 \cdot e^{0.45M} \cdot e^{-0.005D}$$

Where:

a_{V2} = acceleration of Variant 2 in Gal;

M = magnitude;

D = epicentral distance in km.

$V2$ increases the scale factor and reduces the direct exponential sensitivity to magnitude.

6.4 Reduced Original

$$a_{\text{Red}} = 18 \cdot e^{0.52M} \cdot e^{-0.005D}$$

Where:

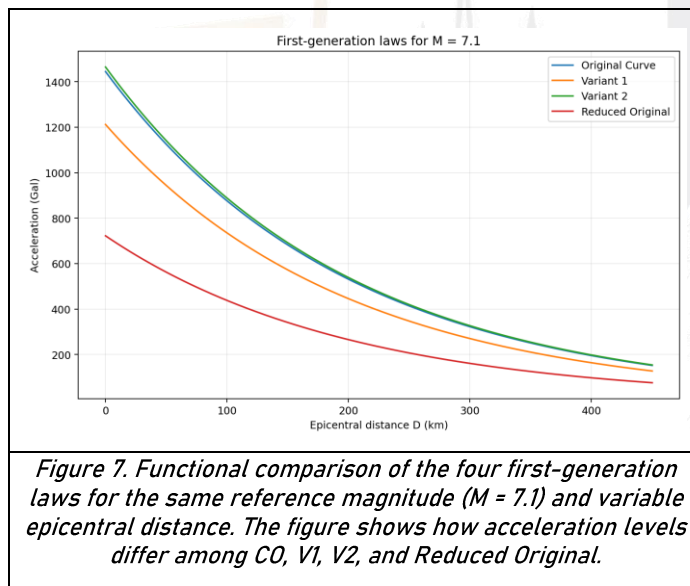
a_{Red} = acceleration of the Reduced Original in Gal;

M = magnitude;

D = epicentral distance in km.

The Reduced Original Curve retains the same magnitude–distance dependence as the Original Curve. It was proposed for designers who wish to reduce the mathematical severity of the attenuation formulas.

7. Subsequent validation of the laws: summary of the Ramón Casquete and Zamora (2023) thesis



The Moncayo Theurer family of laws was subsequently subjected to independent academic validation in the undergraduate thesis “Efficiency of Attenuation Laws Against Real Earthquakes,” developed at the University of Guayaquil by Adriana Lissette Ramón Casquete and Ingrid Zamora in 2023 [31].

The study substantially expanded the comparison domain: it selected 50 control earthquakes distributed across different magnitude ranges, with a minimum of approximately 20 records per event, and evaluated 18

formulations: 13 historical laws and the five equations of the Moncayo Theurer family.

The thesis compared predictions with recorded accelerations and with the CMAR of each event, and used error percentages and a decision matrix to rank performance.

Overall, the results supported the operational validity of the Moncayo Theurer formulations.

The thesis showed that there is no single dominant formulation across the entire magnitude range. For events between Mw 4 and 6, the Second-Generation Law and GOT2 reached a technical tie for first place in the overall evaluation.

For magnitudes from Mw 7 upward, GOT2 and Variant 1 showed the best aggregate performance according to the decision matrix.

The Second-Generation attenuation law showed strong performance for large earthquakes: for example, in Pedernales Mw 7.8 and Chile Mw 8.8 it produced PGA errors on the order of 3% and 8%, respectively [31].

The Original Curve is a simple and effective formulation that reproduces high demand near the epicenter well and constitutes the mathematical basis of the first generation.

The Second-Generation Law retains that philosophy, but corrects a structural limitation of the first equations: the use of a fixed spatial attenuation rate.

Five cases studied in detail illustrate this conclusion: in California Mw 4.7, the Second Generation achieved approximately 2% error relative to PGA; in Alaska Mw 6.1 it obtained about 22% compared with 29% for GOT2; in Pedernales, Ecuador, Mw 7.8 the Second Generation approximated PGA with an error close to 3%, while GOT2 showed good performance relative to CMAR; and in Chile Mw 8.8, Variant 2 produced approximately 7% error relative to CMAR and the Second Generation about 8% relative to PGA [31].

The main value of that thesis is that it provides independent evidence obtained after development of the four first-generation laws. The validation supports the view that the proposed family has competitive

predictive performance when compared with real-earthquake records and that, over specific ranges, Variant 1 and Variant 2 achieve favorable results relative to historical formulations.

8. Conclusions

1. First-generation THEK-RRAM was developed from real instrumental records representing four magnitude levels: Chino Hills Mw 5.4, Northridge Mw 6.4, Miyagi Mw 7.1, and Tohoku Mw 9.0.
2. CMAR constitutes the envelope of the maximum recorded accelerations for each event and serves as the upper empirical reference for development; it is not an average curve.
3. The sequence currently formalized as FGET-VRR-NFF describes the transition from observing acceleration-distance patterns to an explicit first-generation structure, without requiring publication of all the operational know-how used in the internal iterations.
4. The Ramón Casquete and Zamora (2023) thesis, based on 50 control earthquakes, provides subsequent independent validation and shows that the Moncayo Theurer formulations remain competitive across different magnitude ranges, with particularly favorable results for Variant 1, Variant 2, and, in the full study, the Second Generation.

Note on application of the method

For information on full reproduction, internal criteria, and implementation, contact research@hithek.com.

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